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REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
<small>Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of the collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.</small>				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE		3. REPORT TYPE AND DATES COVERED
				Final Report, 15 Jun 86 to 14 Sep 89
4. TITLE AND SUBTITLE NUMERICAL ANALYSIS			5. FUNDING NUMBERS AFOSR-86-0163 61102F 2304/A3	
6. AUTHOR(S) Seymour V. Parter				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Computer Sciences Department University of Wisconsin Madison, WI 53706			8. PERFORMING ORGANIZATION REPORT NUMBER  AFOSR-TR-90-0550	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFOSR/NM Bldg 410 Bolling AFB DC 20332-8448			10. SPONSORING/MONITORING AGENCY REPORT NUMBER  AFOSR-86-0163	
11. SUPPLEMENTARY NOTES  DTIC ELECTE MAY 29 1990				
12a. DISTRIBUTION/AVAILABILITY STATEMENT  Approved for public release; distribution unlimited.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  → The proposal to AFOSR emphasized research on the effective solution of discrete elliptic equations. In particular the proposal emphasized that the research would concentrate on four main areas: 1) Sharp Estimates for Multigrid Methods; 2) Preconditioned Conjugate Gradient methods with a particular emphasis on multigrid as a preconditioner; 3) Indefinite elliptic problems and singularly perturbed elliptic problems; 4) The implementation of these methods on parallel computers and multiprocessors. Significant progress was made on all the above areas with the exception of topic 4. The press of preliminary theoretical work and computational studies undertaken in support of that theoretical work just kept us too busy. (KR) ←				
14. SUBJECT TERMS			15. NUMBER OF PAGES 7	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT MUNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT SAR	

**Final Scientific Report**  
**on**  
**Air Force Office of Scientific Research**

**Contract No. AFSOR-86-0163**

**June 15, 1986 - September 14, 1989**



Accession For	
NTIS CRA&I	<input checked="checked" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By _____	
Distribution /	
Availability Codes	
Dist	Avail and/or Special
A-1	

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## I. Research Objectives and Status

A. Proposed Research Goals The proposal to AFOSR emphasized research on the effective solution of discrete elliptic equations. In particular the proposal emphasized that the research would concentrate on four main areas.

1. Sharp Estimates for Multigrid Methods.
2. Preconditioned Conjugate Gradient methods with a particular emphasis on multigrid as a preconditioner.
3. Indefinite elliptic problems and singularly perturbed elliptic problems.
4. The implementation of these methods on parallel computers and multiprocessors.

B. Status of the Research: Significant progress was made on all the above areas with the exception of topic 4. The press of preliminary theoretical work and computational studies undertaken in support of that theoretical work just kept us too busy.

1. In the area of sharp estimates for Multigrid Methods there were three significant papers.

(a) S.V. Parter, Estimates for Multigrid Methods Based on Red-Black Gauss-Seidel Smoothings, Numerische Mathematik, 52, 701-723 (1988). In this paper precise numerical values were obtained for a large class of multigrid methods. These red-black Gauss-Seidel smoothing methods are among the most frequently used multigrid methods. They are particularly attractive for use on parallel processors.

(b) N.H. Decker, J. Mandel and S.V. Parter, On the role of regularity in multigrid methods. Multigrid Methods, Theory Applications and Supercomputing, Edited by S.F. McCormick, pp. 143-156, Marcel Dekker, Inc., 1988 [Proceedings of the Third Copper Mountain Multigrid Conference].

(c) J. Mandel and S.V. Parter, On the Multigrid F-Cycle. To Appear: Applied Mathematics and Computation.

Both of these papers are concerned with estimating the "rate of convergence" of multigrid methods (the V-cycle and the F-cycle) in the case where full  $H_2$  elliptic regularity is not present. When there is such  $H_2$  elliptic regularity these multigrid methods possess a "rate of convergence" which is independent of the number of grids. Without such regularity experimental evidence has shown that there is a decay in this rate of convergence as the number of grids increases. These papers provide asymptotic estimates of this behavior. The results may not be sharp, but the bounds provide valuable practical insight in how one should proceed.

Problems without  $H_2$  elliptic regularity occur often in practice. Hence these studies are particularly relevant for the practical implementation of multigrid.

2. In the area of Preconditioned Conjugate Gradient with an emphasis on Multigrid as a Preconditioner there were several very important contributions.

The thesis work of Ms. Naomi Decker was aimed at the study of a particular multigrid preconditioner (suggested in a Brookhaven National Laboratory Report by C.I. Goldstein) for singular perturbation problems. Therefore, her work combined our interest in topics 2 and 3. These results appeared in:

- (a) The k-Grid Fourier Analysis of Multigrid-Type Iterative Methods, CS Technical Report #703, University of Wisconsin
- (b) The Fourier Analysis of Multigrid-Type Iterative Methods, Thesis, August 1987
- (c) The Fourier Analysis of a Multigrid Preconditioner in Multigrid Methods, S.F. McCormick, ed., vol. 110 of Lecture Notes in Pure and Applied Mathematics, 1987, Marcel Dekker, New York.

In the general area of preconditioning there are three lengthy reports. Two of which have been accepted for publication and will appear shortly. We have every reason to believe the third will also be accepted for publication. These reports represent several breakthroughs on the general topic: Given an elliptic operator  $A$  and a discretization  $A_h$ , how does one determine appropriate choices of other elliptic operators  $B$ , with discretizations  $B_h$ , so that the preconditioned operators  $B_h^{-1}A_h, A_hB_h^{-1}$  are well conditioned. In these works we have not only studied the necessary and sufficient conditions that

$$\| B_h^{-1}A_h \|_{L_2}, \quad \| A_hB_h^{-1} \|_{L_2}, \quad \| B_h^{-1}A_h \|_{H_1}$$

be uniformly bounded, independent of  $h$ , but we have also begun the study of the distribution of the singular values ( $L_2$ ) of  $B_h^{-1}A_h, A_hB_h^{-1}$ . This last topic has usually been considered too delicate and untactable. Previous research has concentrated almost entirely on the first question of  $\| B_h^{-1}A_h \|_{L_2}, \| A_hB_h^{-1} \|_{L_2}$ , and  $\| B_h^{-1}A_h \|_{H_1}$ . Indeed, most earlier work has been limited to the  $H_1$  case. Unlike previous reports (by other authors) on this question we have not required the definiteness of either  $A$  or  $B$ . Thus, once more, this work also has implications for Area (3). Further, the basic results carry over when  $B_h^{-1}$  is replaced by  $\hat{B}_h^{-1}$ , a multigrid cycle for  $B_h^{-1}$ . This is very important because in practice  $B_h^{-1}$  is extremely difficult to obtain while  $\hat{B}_h^{-1}$  is quite accessible in many cases (e.g. when  $B$  is definite).

The three reports are:

- (a) V. Faber, T.A. Manteuffel and S.V. Parter, On the Equivalence of Operators and the Implications to Preconditioned Iterative Methods for Elliptic Problems, Los Alamos National Laboratory report LA-UR-86-2152. To Appear: Advances in Applied Mathematics.
  - (b) T.A. Manteuffel and S.V. Parter, "Preconditioning and Boundary Conditions," Los Alamos National Laboratory report LA-UR-88-2626, To Appear: SIAM Journal Numerical Analysis.
  - (c) W. Joubert, T.A. Manteuffel, S.V. Parter and Sze-Ping Wong, "Preconditioning of Second Order Elliptic Operators: Experiment and Theory," Preliminary Proceedings Copper Mountain Conference on Iterative Methods, April 1-5, 1990.
3. In the Area of Indefinite Elliptic problems and singularly perturbed elliptic problems there have been two papers--in addition to the work mentioned above which also impinges on this area. These are:
- (a) S. V. Parter, "Preconditioning for Singular Perturbation Problems". Proceedings of the Fourth International Conference on Boundary and Interior Layers--Computational and Asymptotic Methods, July 7-11, 1986. NOVOSIBIRSK, USSR. Edited by S.K. Godunov, J.J.H. Miller and V.A. Novikov. pp. 403-411, Boole Press, Dublin (1986).
  - (b) C.I. Goldstein and S.V. Parter, "On the Norm Equivalence of Singularly Perturbed Elliptic Difference Operators," Applied Math. and Comp. 26, 179-200 (1988).

There have been several other publications. However, the above discussion is limited to the most important results.

## II. Interactions

During the period of this grant, Seymour Parter, the Principal Investigator has given many talks on this research and collaborated with individuals in several other institutions. e.g. the Computational Mathematics Group at the University of Colorado at Denver, the Los Alamos National Laboratory and the Brookhaven National Laboratory.

The following is a list of most of the Professional Lectures given by the personnel supported under this grant during the grant period.

Naomi H. Decker:

"The Fourier Analysis of a Multigrid Preconditioner" Lecture presented at the 3rd Copper Mountain Multigrid Conference, April 5-10, 1987.

Seymour V. Parter:

October 6, 1986, Northwestern University, Applied Math. Colloquium; "Another Look at the Rotating Disk Problem"

October 7, 1986, University of Chicago, Applied Math Seminar, "Another Look at the Rotating Disk Problem"

July 1986 - Fourth International Conference on Boundary and Interior Layers-Computational and Asymptotic Methods, Novosibirsk, USSR, "Preconditioning for Singular Perturbation Problems".

December 1986-Applied Math Seminar Tel-Aviv University, Tel-Aviv, Israel. "The Singular Values of Toeplitz Matrices"

January 1987-2nd Haifa Matrix Conference, Haifa, Israel, "The Singular Values of Toeplitz Matrices".

April 1987-Third Copper Mountain Multigrid Conference, "On the Role of Regularity in Multigrid Methods"

August 1987-Second International Symposium on Numerical Analysis-Prague, Czechoslovakia. Invited Plenary Lecture, "Remarks on the Inversion of Toeplitz Matrices"

October 1987-Oberwolfache Conference on Multigrid, "On the Role of Regularity in Multigrid Convergence Theorems."

October 1987-Applied Mathematics Seminar, Brown University, "Preconditioning and Boundary Conditions."

November 1987-Applied Mathematics Colloquium, Columbia University, "Multigrid Convergence."

January 1988-Applied Mathematics Seminar, Tel-Aviv University, Tel-Aviv, Israel, "Preconditioning and Boundary Conditions."

September 1988-Seminar at Center for Supercomputing Research, University of Illinois. "Preconditioning and Boundary Conditions"

January 1989-Fifth Haifa Matrix Theory Conference, Haifa, Israel, "Preconditioning and Boundary Conditions."

February 1989-Applied Mathematics Seminar, University of Maryland, College Park, MD. "Preconditioning and Boundary Conditions."

July 1989—International Conference on High Order Numerical Methods for Partial Differential Equations, Como, Italy, "Preconditioning and Boundary Conditions: Spectral Problems."

July 1989—Annual SIAM meeting, San Diego, California, Invited Mini-Symposium Speaker. "On the Theory of Equivalent Operators and Application to the Numerical Solution of Uniformly Elliptic Partial Differential Equations."

### III. Publications

The following papers, reports, and theses were prepared with the support or partial support of this Grant.

D. Kamowitz: Theoretical and Experimental Results for a Variety of Multigrid Algorithms. Ph.D. thesis, Computer Sciences Department, August 1986.

S. V. Parter, "Preconditioning for Singular Perturbation Problems" Proceedings of the Fourth International Conference on Boundary and Interior Layers—Computational and Asymptotic Methods, July 7-11, 1986, NOVOSIBURSK, USSR. Edited by S. K. Godunov, J.J.H. Miller and V. A. Novikov, pp. 403-411, Boole Press, Dublin (1986).

C. I. Goldstein and S. V. Parter, "On the Normal Equivalence of Singularly Perturbed Elliptic Difference Operators," Applied Math. and Comp. 26, 179-200 (1988).

N. H. Decker, J. Mandel and S. V. Parter, On the role of regularity in multigrid methods. Multigrid Methods, Theory Applications and Supercomputing, Edited by S. F. McCormick, pp. 143-156, Marcel Dekker, Inc., 1988 [Proceedings of the Third Copper Mountain Multigrid Conference.]

J. Mandel and S. V. Parter, On the Multigrid F-Cycle. To Appear: Applied Mathematics and Computation.

V. Faber, T. A. Manteuffel and S. V. Parter, On the Equivalence of Operators and the Implications to Preconditioned Iterative Methods for Elliptic Problems, Los Alamos National Laboratory report LA-UR-86-2152. To Appear: Advances in Applied Mathematics.

T. A. Manteuffel and S. V. Parter, "Preconditioning and Boundary Conditions." Los Alamos National Laboratory report LA-UR-88-2626. To Appear: SIAM Journal Numerical Analysis.

W. Joubert, T. A. Manteuffel, S. V. Parter and Sze-Ping Wong, "Preconditioning of Second Order Elliptic Operators: Experiment and Theory," Preliminary Proceedings Copper Mountain Conference on Iterative Methods, April 1-5, 1990.

S. V. Parter, "Remarks on the Solution of Toeplitz Systems of Equations" Proceedings 2nd International Symposium on Numerical Analysis--Prague, Czechoslovakia. Aug. 1987. pp. 75-81, Teubner-Texte Zur Mathematik, Band 107.

S. V. Parter, Estimates for Multigrid Methods Based on Red-Black Gauss-Seidel Smoothings. Numerische Mathematik, 52, 701-723 (1988).

S.V. Parter, "Remarks on Multigrid Convergence Theorems" Appl. Math. and Comp. 23: 103-120 (1987)

N. H. Decker. "The K-Grid Fourier Analysis of Multigrid-Type Methods" CS Department Technical Report #703, University of Wisconsin, July 1987.

N. H. Decker. The Fourier Analysis of Multigrid-Type Iterative Methods. Ph.D. Thesis. Department of Mathematics, August 1987.

N. H. Decker. "The Fourier Analysis of a Multigrid Preconditioner in Multigrid Methods." S. F. McCormick, ed., vol. 110 of Lecture Notes in Pure and Applied Mathematics. 1987. Marcel Dekker. New York.

#### IV. Personnel

The following individuals were supported by this grant.

Seymour V. Parter (Principal Investigator)

David Kamowitz (Research Assistant) Ph.D. in Computer Science awarded August 1986. Thesis title: Theoretical and Experimental Results for a Variety of Multigrid Algorithms.

Naomi H. Decker (Research Assistant) Ph.D. in Mathematics awarded August 1987. Thesis Title: The Fourier Analysis of Multigrid-Type Iterative Methods.

Sze-Ping Wong (Research Assistant) A graduate student in the Department of Mathematics. His research work is concerned with preconditioning for Elliptic Boundary Value problems.